

## More Extremal Problems

1. (a) Use Lagrange multipliers to find the absolute minimum and maximum values of  $f(x, y) = x^2 + 4y^2$  subject to the constraint  $y = x^2 - 2$ , if they exist.

- (b) Sketch the level set diagram of  $f(x, y) = x^2 + 4y^2$  and the constraint curve  $y = x^2 - 2$ . Where are the candidate points that the method of Lagrange multipliers finds?

2. Decide whether each statement is true or false. (If true, explain what strategy you would use to find the absolute minimum and maximum values.)
- (a) Every continuous function  $f(x, y)$  must attain an absolute minimum and absolute maximum value on  $x^2 + 4y^2 < 1$ .
  - (b) Every continuous function  $f(x, y)$  must attain an absolute minimum and absolute maximum value on  $x^2 + 4y^2 = 1$ .
  - (c) Every continuous function  $f(x, y)$  must attain an absolute minimum and absolute maximum value on  $x^2 - 4y^2 = 1$ .
  - (d) Every continuous function  $f(x, y)$  must attain an absolute minimum and absolute maximum value on  $x^2 + 4y^2 \leq 1, y \geq 0$ .
  - (e) Every continuous function  $f(x, y, z)$  must attain an absolute minimum and absolute maximum value on  $x^2 + 4y^2 + z^2 = 1$ .
  - (f) Every continuous function  $f(x, y, z)$  must attain an absolute minimum and absolute maximum value on  $x^2 + 4y^2 = 1$ .
  - (g) Every continuous function  $f(x, y, z)$  must attain an absolute minimum and absolute maximum value on the intersection of  $x^2 + 4y^2 + z^2 = 1$  and  $x + y + z = 1$ .

3. Find the absolute maximum and minimum values of  $xyz^2$  on the solid  $x^2 + 4y^2 + z^2 \leq 16$ , if they exist.